

Relativistic tidal heating of Hamiltonian quasi-local boundary expressions

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Abstract

Purdue and Favata calculate the tidal heating used certain classical pseudotensors. Booth and Creighton employed the quasi-local mass formalism of Brown and York to demonstrate the same subject. All of them give the result matched with the Newtonian theory. Here we present another Hamiltonian quasi-local boundary expressions and all give the same desired value. This indicates that the tidal heating is unique as Thorne predicted. Moreover, we discovered that the pseudo-tensor method and quasi-local method are fundamentally different.

1 Introduction

In gravitation, one of the interested topics is that calculate the tidal heating: the interaction between a nearly isolated gravitating body and the external universe. Solar system provides a typical example for the illustration: Jupiter and its satellite Io [1]. Purdue [2] and Favata [3] examined the tidal heating for the classical pseudotensors. Booth and Creighton used the quasi-local mass formalism of Brown and York to demonstrate the same subject [4]. All of them give the same value as the Newtonian perspective. Here we present another Hamiltonian quasi-local boundary expressions to examine the tidal heating, we find that the result is unique as Purdue achieved, i.e., quasi-local expressions independent.

Thorne claimed that all pseudo-tensors give the same tidal work as the Newtonian gravity [5]. Nester realized that pseudo-tensor method and quasi-local formalism are basically the same [6], however, Booth and Creighton prefer the quasi-local method such that all quantities can be manipulated in terms of real tensors on the quasi-local surface [4]. Thus one may imagine that there is no surprise using quasi-local expressions to give the same desired tidal work. Although using the pseudo-tensor method and quasi-local method give the same tidal heating rate, we find that the fundamental principle between these two are different. In particular, the Møller pseudo-tensor give the standard tidal heating [3] but failed for inside matter requirement [7]. Meanwhile the Hamiltonian quasi-local method can rehabilitate this handicap.

Generally speaking, different energy-momentum pseudo-tensor refers to different gauge condition for the gravitational energy localization, while different quasi-local boundary expression focus on different boundary condition. Confined to the tidal heating, we claim that the gauge condition and boundary condition are elementarily the same terminology. Different gauge condition corresponds different E_{int} , where E_{int} is the energy interaction between the isolated planet's quadrupolar deformation and the external tidal field. We find that the tidal heating remains unchange for different quasi-local boundary expressions, thus the tidal heating is gauge invariant. However there is a change: the exchangeable energy rate \dot{E}_{int} , they are gauge dependent. Here we explain some terminology used in present paper. The expected tidal heating or tidal work rate $\dot{W} = -\frac{1}{2}\dot{I}_{ij}E^{ij}$, where W refers to tidal work, the dot means differentiate w.r.t. time t , I_{ij} is the mass quadrupole moment of the isolated planet and E_{ij} is the tidal field of the external universe. Both I_{ij} and E_{ij} are time dependent, but symmetric and trace free. Here we emphasize that the tidal heating at which the external field does work on the isolated body and this is an energy dissipation process which means energy unexchangeable. Conversely, there is an exchangeable process

$\dot{E}_{\text{int}} \sim \frac{d}{dt}(I_{ij}E^{ij})$. Purdue uses $E_{\text{int}} = \frac{\beta+2}{10}I_{ij}E^{ij}$ to distinguish different options how to localize the gravitational energy by tuning the coefficient β [2].

2 Technical background

We used the same spacetime signature and notation as in [8]: let the geometrical units $G = c = 1$, where G and c are the Newtonian constant and speed of light. The Greek letters refer to the spacetime and Latin letters indicate the spatial. For the idea of energy-momentum pseudo-tensor $t_\alpha{}^\mu$, choose an appropriate super-potential $U_\alpha{}^{[\mu\nu]}$

$$\partial_\nu U_\alpha{}^{[\mu\nu]} = 2\kappa\sqrt{-g}(T_\alpha{}^\mu + t_\alpha{}^\mu), \quad (1)$$

where $T_\alpha{}^\mu$ is the stress tensor, $\partial_\nu U_\alpha{}^{[\mu\nu]}$ can be described as the total energy-momentum complex and it is conserved since $\partial_{\mu\nu}^2 U_\alpha{}^{[\mu\nu]} \equiv 0$. Recall the Einstein equation: $G_{\mu\nu} = \kappa T_{\mu\nu}$, where $G_{\mu\nu}$ is the Einstein tensor and $\kappa = 8\pi G/c^4$. Look closer at (1), $\partial_\nu U_\alpha{}^{[\mu\nu]}$ consists two parts: $2G_\alpha{}^\mu$ is the mass energy inside matter and $t_\alpha{}^\mu$ is the gravitational energy-momentum in vacuum. The component $t_0{}^j$ is the gravitational energy flux density. The criterion of the interior mass-energy is important. In particular, the classical Møller pseudo-tensor cannot satisfy this inside matter condition [7]. Thus one can conclude that Møller pseudo-tensor is not appropriate to describe the energy-momentum in vacuum.

The gravitational tidal heating rate can be computed as

$$2\kappa\dot{W} = \oint_{\partial V} \sqrt{-g}t_0{}^j\hat{n}_j r^2 d\Omega, \quad (2)$$

where $r \equiv \sqrt{\delta_{ab}x^a x^b}$ is the distance from the body in its local asymptotic rest frame and $\hat{n}_j \equiv x_j/r$ is the unit radial vector. In our calculation, the metric tensor can be decomposed as $g_{\alpha\beta} = \eta_{\alpha\beta} + h_{\alpha\beta}$ and its inverse $g^{\alpha\beta} = \eta^{\alpha\beta} - h^{\alpha\beta}$. We have the following physical expressions [2]:

$$h_{00} = \frac{2M}{r} + \frac{3}{r^5}I_{ab}x^a x^b - E_{ab}x^a x^b, \quad h_{0j} = \frac{2}{r^3}\dot{I}_{ij}x^i - \frac{10}{21}\dot{E}_{ab}x^a x^b x_j + \frac{4}{21}\dot{E}_{ij}x^i r^2, \quad (3)$$

note that $h_{ij} = \delta_{ij}h_{00}$.

3 Hamiltonian quasi-local boundary expressions

Geometric theories are invariant under local diffeomorphism. Here we review the Hamiltonian quasi-local boundary expressions from a first order Lagrangian [9]:

$$\mathcal{L} = dq \wedge p - \Lambda(q, p), \quad (4)$$

where q and p are canonical conjugate form fields, Λ is a potential. Let q be a f -form and $\epsilon = (-1)^f$. The corresponding Hamiltonian 3-form is defined as follows

$$\mathcal{H}(N) := \mathcal{L}_N q \wedge p - i_N \mathcal{L}. \quad (5)$$

Taking the interior product of the Lagrangian density

$$i_N \mathcal{L} = \mathcal{L}_N q \wedge p - \epsilon i_N q \wedge dp - \epsilon dq \wedge i_N p - i_N \Lambda - d(i_N q \wedge p), \quad (6)$$

where the Lie derivative $\mathcal{L}_N = i_N d + di_N$. Using (6), rewrite (5)

$$\mathcal{H}(N) = N^\mu \mathcal{H}_\mu + d\mathcal{B}(N), \quad (7)$$

where $N^\mu \mathcal{H}_\mu = \epsilon i_N q \wedge dp + \epsilon dq \wedge i_N p + i_N \Lambda$ which is proportional to the field equations and vanishes ‘on shell’. Note that N^μ is the vector field. The Hamiltonian density \mathcal{H}_μ determines the evolution equations and initial value constraints. The natural boundary term $\mathcal{B}(N) = i_N q \wedge p$. However, this boundary term is not unique since it can be removed by introducing a new Hamiltonian

$$\mathcal{H}'(N) = \mathcal{H}(N) + d(-i_N q \wedge p) = N^\mu \mathcal{H}_\mu. \quad (8)$$

Taking the variation of this new Hamiltonian

$$\delta \mathcal{H}'(N) = -i_N(\text{F.E.}) - \delta q \wedge \mathcal{L}_N p + \mathcal{L}_N q \wedge \delta p + d\mathcal{B}(N), \quad (9)$$

where the field equation $\text{F.E.} = \delta q \wedge \frac{\delta \mathcal{L}}{\delta q} + \frac{\delta \mathcal{L}}{\delta p} \wedge \delta p$. The boundary variation term is

$$\mathcal{B}(N) = -i_N q \wedge \delta p + \epsilon \delta q \wedge i_N p. \quad (10)$$

This $\mathcal{B}(N)$ cannot be removed because it comes from $\delta \mathcal{H}'$ directly. Boundary conditions can be obtained through the boundary term in the variation of the Hamiltonian vanishes. We add an appropriate boundary term to the Hamiltonian

$$\mathcal{H}'(N) \rightarrow \mathcal{H}_k(N) = N^\mu \mathcal{H}_\mu + d\mathcal{B}_k(N), \quad (11)$$

to modify the variational boundary term. In order to achieve nice components like $i_N(\delta q \wedge \Delta p)$ or $i_N(\Delta q \wedge \delta p)$, there are four simple boundary expressions can be added. The variation of the four Hamiltonians including this four expressions are

$$\delta \mathcal{H}_q(N) = K + di_N(\delta q \wedge \Delta p), \quad \delta \mathcal{H}_d(N) = K - d(i_N \Delta q \wedge \delta p - \epsilon \delta q \wedge i_N \Delta p), \quad (12)$$

$$\delta \mathcal{H}_p(N) = K - di_N(\Delta q \wedge \delta p), \quad \delta \mathcal{H}_c(N) = K + d(i_N \delta q \wedge \Delta p - \epsilon \Delta q \wedge i_N \delta p), \quad (13)$$

where $K = -i_N(\text{F.E.}) - \delta q \wedge \mathcal{L}_N p + \mathcal{L}_N q \wedge \delta p$. Thus we recovered [6]

$$\mathcal{B}_q(N) = i_N q \wedge \Delta p - \epsilon \Delta q \wedge i_N \bar{p}, \quad \mathcal{B}_d(N) = i_N \bar{q} \wedge \Delta p - \epsilon \Delta q \wedge i_N \bar{p}, \quad (14)$$

$$\mathcal{B}_p(N) = i_N \bar{q} \wedge \Delta p - \epsilon \Delta q \wedge i_N \bar{p}, \quad \mathcal{B}_c(N) = i_N q \wedge \Delta p - \epsilon \Delta q \wedge i_N \bar{p}, \quad (15)$$

where $\Delta q = q - \bar{q}$, $\Delta p = p - \bar{p}$, both \bar{q} and \bar{p} are the background reference values. Alternatively, rewrite the above four equations in a compact form

$$\mathcal{B}_{k_1, k_2}(N) = \mathcal{B}_p(N) + k_1 i_N \Delta q \wedge \Delta p + \epsilon k_2 \Delta q \wedge i_N \Delta p, \quad (16)$$

where k_1 and k_2 can be 0 or 1. In detail $\mathcal{B}_{0,0} = \mathcal{B}_p$, $\mathcal{B}_{0,1} = \mathcal{B}_d$, $\mathcal{B}_{1,0} = \mathcal{B}_c$ and $\mathcal{B}_{1,1} = \mathcal{B}_q$.

Using the analogy of classical electrodynamics and apply to the relativistic gravity, the type of boundary condition should be either Dirichlet or Neumann, and even a mixture of these two. From the boundary condition point of view, we prefer \mathcal{B}_q and \mathcal{B}_p since they are the simplest, i.e., Dirichlet or Neumann. Meanwhile, the boundary conditions of \mathcal{B}_c or \mathcal{B}_d could be a certain linear combination of Dirichlet and Neumann. For the case of \mathcal{B}_q , there are two ways to obtain the variation of the boundary term that satisfy $i_N(\delta q \wedge \Delta p) = 0$. First control q , then $\delta q = 0$. The second is to freely vary q and then δq becomes arbitrary, which implies $\Delta p = 0$. This is the ‘natural boundary condition’ as it forces $p = \bar{p}$. Similarly for the variation boundary term $i_N(\Delta q \wedge \delta p) = 0$. In addition, we have modified $(k_1, k_2) \rightarrow (c_1, c_2)$, where c_1 and c_2 are arbitrary constants, such that the variation is still legitimate [9].

3.1 Quasi-local Møller and Freud super-potentials

Here we apply this Hamiltonian formalism to the Einstein-Hilbert Lagrangian [6]

$$\mathcal{L}_{\text{GR}} := R^\alpha{}_\beta \wedge \eta_\alpha{}^\beta, \quad (17)$$

where the curvature 2-form $R^\alpha{}_\beta = d\Gamma^\alpha{}_\beta + \Gamma^\alpha{}_\gamma \wedge \Gamma^\gamma{}_\beta$, the connection 1-form $\Gamma^\alpha{}_\beta = \Gamma^\alpha{}_{\beta\gamma} dx^\gamma$ and the dual basis is $\eta^{\alpha\beta\cdots} = *(dx^\alpha \wedge dx^\beta \cdots)$. As before, the interior product

$$i_N \mathcal{L} = i_N R^\alpha{}_\beta \wedge \eta_\alpha{}^\beta + R^\alpha{}_\beta \wedge i_N \eta_\alpha{}^\beta = \mathcal{L}_N \Gamma^\alpha{}_\beta \wedge \eta_\alpha{}^\beta - \mathcal{H}(N). \quad (18)$$

After a straightforward manipulation, the Hamiltonian density from above becomes

$$\mathcal{H}(N) = N^\mu \mathcal{H}_\mu - i_N \Gamma^\alpha{}_\beta D\eta_\alpha{}^\beta + d\mathcal{B}(N), \quad (19)$$

where $\mathcal{H}_\alpha = 2G^\rho{}_\alpha \eta_\rho$ which satisfies the dynamical evolution and initial value constraints, $i_N \Gamma^\mu{}_\nu \wedge D\eta_\mu{}^\nu$ vanishes since the metric compatibility. Finally the boundary term $\mathcal{B} = i_N \Gamma^\mu{}_\nu \wedge \eta_\mu{}^\nu = N^\alpha g^{\nu\sigma} \Gamma^\mu{}_{\sigma\alpha} \eta_{\mu\nu}$. Note that it is legal to modify the boundary term replace a negative sign, i.e., $\mathcal{B} \rightarrow -\mathcal{B}$. Rewrite (19)

$$\mathcal{H}(N) = N^\alpha \left[2G^\rho{}_\alpha \eta_\rho - \frac{1}{2} \sqrt{-g} (g^{\nu\sigma} \Gamma^\mu{}_{\sigma\alpha} - g^{\mu\sigma} \Gamma^\nu{}_{\sigma\alpha}) dS_{\mu\nu} \right], \quad (20)$$

where $dS_{\mu\nu} = \frac{1}{2} \epsilon_{\mu\nu\xi\kappa} dx^\xi \wedge dx^\kappa$. Looking at (20), the term ${}_M \mathcal{U}_\alpha^{[\mu\nu]} = 2\sqrt{-g} g^{\sigma[\mu} \Gamma^{\nu]}{}_{\sigma\alpha}$ looks like Møller super-potential [10] but not exactly. Instead we call this quasi-local Møller super-potential. The reason is that the Møller pseudo-tensor cannot fulfill the interior mass-energy requirement $2G^\rho{}_\mu$ as explained before, but the quasi-local Møller super-potential ${}_M \mathcal{U}_\alpha^{[\mu\nu]}$ can. The variation of (19)

$$\delta \mathcal{H} = \text{field equation terms} + di_N (\delta \Gamma^\alpha{}_\beta \wedge \eta_\alpha{}^\beta). \quad (21)$$

This can be classified as Neumann type boundary condition, i.e., the connection $\Gamma \simeq \partial g$ is to be held fixed. Alternatively this terminology can be translated as the deDonder gauge: $0 = \partial_\kappa (\sqrt{-g} g^{\xi\kappa}) = \sqrt{-g} \Gamma^{\xi\kappa}{}_\kappa$. Using the analogy of the pseudo-tensor method, the quasi-local Møller pseudo-tensor is ${}_M \mathbf{t}_\alpha{}^\mu = \partial_\nu ({}_M \mathcal{U}_\alpha^{[\mu\nu]})$. It is known that the tidal heating is $\dot{W}_M = -\frac{1}{2} \dot{I}_{ij} E^{ij}$, where $\dot{E}_{\text{int}} = \frac{\beta+2}{10} \frac{d}{dt} (I_{ij} E^{ij}) = 0$ which means the energy localization chosen $\beta = -2$ [3].

Keep the same track in [6], swap the quasi-local Møller super-potential to the other pattern $\mathcal{B}' = \Gamma^\mu{}_\nu \wedge i_N \eta_\mu{}^\nu = \frac{1}{2} N^\alpha g^{\beta\sigma} \Gamma^\lambda{}_{\beta\gamma} \delta_{\lambda\sigma\alpha}^{\gamma\mu\nu} \eta_{\mu\nu}$. Again flip the sign of \mathcal{B}' and rewrite (19)

$$\mathcal{H}(N) = N^\alpha \left[2G^\rho{}_\alpha \eta_\rho - \frac{1}{2} \sqrt{-g} g^{\beta\sigma} \Gamma^\lambda{}_{\beta\gamma} \delta_{\lambda\sigma\alpha}^{\gamma\mu\nu} dS_{\mu\nu} \right], \quad (22)$$

where ${}_F \mathcal{U}_\alpha^{[\mu\nu]} = -\sqrt{-g} g^{\beta\sigma} \Gamma^\lambda{}_{\beta\gamma} \delta_{\lambda\sigma\alpha}^{\gamma\mu\nu}$ looks like the Freud super-potential [11] but we call this the quasi-local Freud super-potential because it comes from the Hamiltonian formalism. The quasi-local Einstein pseudotensor is ${}_E \mathbf{t}_\alpha{}^\mu = \partial_\nu ({}_F \mathcal{U}_\alpha^{[\mu\nu]})$. The known result for the tidal heating $\dot{W}_E = \frac{3}{10} \frac{d}{dt} (I_{ij} E^{ij}) - \frac{1}{2} \dot{I}_{ij} E^{ij}$, where $\dot{E}_{\text{int}} = \frac{\beta+2}{10} \frac{d}{dt} (I_{ij} E^{ij}) = 0$ which means the energy localization selected $\beta = 1$ [3]. Note that this boundary condition can be described as the Dirichlet type which means fixing $\sqrt{-g} g^{\beta\sigma}$.

Based on the pseudo-tensor method, Thorne claimed that the tidal heating is unique [5] and Purdue verified that indeed it is gauge-invariant [2]. As far as the tidal heating is concerned, we find that the gauge condition and boundary condition are equivalent. Moreover we are going to verify that all the quasi-local boundary expressions obtain the standard tidal heating rate.

3.2 Relativistic quasi-local boundary expressions

Here we write the modified quasi-local expressions in holonomic frames [9]

$$\begin{aligned}\mathcal{B}(N) &= \mathcal{B}_p(N) + c_1 i_N \Delta \Gamma^\alpha{}_\beta \wedge \Delta \eta_\alpha{}^\beta - c_2 \Delta \Gamma^\alpha{}_\beta \wedge i_N \Delta \eta_\alpha{}^\beta \\ &= -\frac{1}{2} N^\alpha \left({}_F\mathcal{U}_\alpha{}^{[\mu\nu]} + c_1 \sqrt{-g} h^{\lambda\pi} \Gamma^\sigma{}_{\alpha\pi} \delta_{\lambda\sigma}^{\mu\nu} + c_2 \sqrt{-g} h^{\beta\sigma} \Gamma^\tau{}_{\lambda\beta} \delta_{\tau\sigma\alpha}^{\lambda\mu\nu} \right) \epsilon_{\mu\nu},\end{aligned}\quad (23)$$

where $\Delta \Gamma^\alpha{}_{\beta\mu} = \Gamma^\alpha{}_{\beta\mu} - \bar{\Gamma}^\alpha{}_{\beta\mu}$, c_1, c_2 are real and finite. For simplicity, consider the reference for flat spacetime $\bar{\Gamma}^\alpha{}_{\beta\mu} = 0$ in Cartesian coordinates. Extract the quasi-local superpotential in (23)

$$\mathcal{U}_\alpha{}^{[\mu\nu]} = {}_F\mathcal{U}_\alpha{}^{[\mu\nu]} + c_1 \sqrt{-g} h^{\lambda\pi} \Gamma^\sigma{}_{\alpha\pi} \delta_{\lambda\sigma}^{\mu\nu} + c_2 \sqrt{-g} h^{\beta\sigma} \Gamma^\tau{}_{\lambda\beta} \delta_{\tau\sigma\alpha}^{\lambda\mu\nu} \quad (24)$$

Bear in mind that the Hamiltonian has already fulfilled the inside matter value $2G_\alpha{}^\beta$. This quasi-local Freud super-potential and the extra higher order terms $h\Gamma$ only contribute the energy-momentum in vacuum. Carry on the calculation and we have the quasi-local pseudo-tensor

$$\begin{aligned}\mathbf{t}_\alpha{}^\mu &= {}_E\mathbf{t}_\alpha{}^\mu + c_1 [(\Gamma^{\lambda\pi}{}_\nu + \Gamma^{\pi\lambda}{}_\nu) \Gamma^\sigma{}_{\alpha\pi} + h^{\lambda\pi} \Gamma^\sigma{}_{\alpha\pi,\nu}] \delta_{\lambda\sigma}^{\mu\nu} \\ &\quad + c_2 [(\Gamma^{\beta\sigma}{}_\nu + \Gamma^{\sigma\beta}{}_\nu) \Gamma^\tau{}_{\lambda\beta} + h^{\beta\sigma} \Gamma^\tau{}_{\lambda\beta,\nu}] \delta_{\tau\sigma\alpha}^{\lambda\mu\nu} \\ &= \delta_\alpha^\mu (\Gamma^{\beta\nu}{}_\lambda \Gamma^\lambda{}_{\beta\nu} - \Gamma^\pi{}_{\pi\nu} \Gamma^{\nu\beta}{}_\beta) + \Gamma^\beta{}_{\beta\alpha} (\Gamma^{\mu\nu}{}_\nu - \Gamma^{\nu\mu}{}_\nu) + (\Gamma^{\beta\mu}{}_\alpha + \Gamma^{\mu\beta}{}_\alpha) \Gamma^\nu{}_{\nu\beta} - 2\Gamma^{\beta\nu}{}_\alpha \Gamma^\mu{}_{\beta\nu} \\ &\quad + c_1 \left[\Gamma^{\lambda\beta}{}_\alpha \Gamma^\mu{}_{\lambda\beta} + \Gamma^{\beta\nu}{}_\alpha \Gamma_{\nu\beta}{}^\mu - \Gamma^\mu{}_{\alpha\beta} \Gamma^{\nu\beta}{}_\nu - \Gamma^\mu{}_{\alpha\pi} \Gamma^{\pi\nu}{}_\nu + h^{\mu\pi} \Gamma^\nu{}_{\alpha\pi,\nu} - h^{\pi\nu} \Gamma^\mu{}_{\alpha\pi,\nu} \right] \\ &\quad + c_2 \left[\begin{aligned} &\delta_\alpha^\mu (2\Gamma^{\beta\nu}{}_\lambda \Gamma^\lambda{}_{\beta\nu} - \Gamma^\beta{}_{\beta\lambda} \Gamma^{\lambda\nu}{}_\nu - \Gamma^\beta{}_{\beta\lambda} \Gamma^{\nu\lambda}{}_\nu) + (2\Gamma^{\mu\beta}{}_\alpha + \Gamma^{\beta\mu}{}_\alpha) \Gamma^\nu{}_{\nu\beta} \\ &- \Gamma^{\beta\nu}{}_\alpha (3\Gamma^\mu{}_{\beta\nu} + \Gamma_{\nu\beta}{}^\mu) + \Gamma^\mu{}_{\alpha\beta} \Gamma^{\beta\nu}{}_\nu - h^{\beta\nu} R^\mu{}_{\beta\alpha\nu} \end{aligned} \right].\end{aligned}\quad (25)$$

Consequently the tidal heating is

$$\dot{W} = \frac{3+2c_2}{10} \frac{d}{dt} (I_{ij} E^{ij}) - \frac{1}{2} \dot{I}_{ij} E^{ij}. \quad (26)$$

This shows that the term with c_2 contribute non-vanishing \dot{E}_{int} . In contrast, the term with c_1 contributes nothing. Hence, the tidal heating rate is indeed boundary conditions independent.

4 Conclusion

Purdue and Favata calculate the tidal heating used classical pseudo-tensors. Booth and Creighton employed the quasi-local mass formalism of Brown and York to demonstrate the same subject. All of them give the result matched with the Newtonian gravity. Here we present another Hamiltonian quasi-local boundary expressions and all give the same desired result. This illustrates that the tidal heating is unambiguous since it is gauge invariant. In fact the ambiguity comes from E_{int} : energy interaction between the isolated body quadrupolar deformation and external tidal field.

Meanwhile one may argue that this tidal heating uniqueness is natural because the principle of the pseudo-tensor method and quasi-local method are essentially equivalent. Although the main purpose of these two methods is to evaluate the tidal heating and obtain the same value, we discovered that they are different fundamentally. We prefer the Hamiltonian quasi-local formalism since it guarantees the dynamical evolution and initial value constraints, while the pseudo-tensor method cannot warranty the inside matter requirement.

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